FUNDAMENTAL GROUPS IN ARITHMETIC GEOMETRY 2016 - BIBLIOGRAPHY

Galois categories (J. Stix)

Expected pre-requisites for J. Stix's lectures:

- knowing the topological fundamental group (as a motivation)
- pro-finite groups, absolute Galois groups
- etale maps

A. Grothendieck et al
Revêtements étales et groupe fondamental (SGA1), L.N.M. **224**, Springer-Verlag, 1971.

The 'bible' on Galois categories; it is very rewarding to read and a must for anyone interested in the details. But it's not a survey though...

J.P. Murre
An introduction to Grothendieck's theory of the fundamental group, T.I.F.R., 1967.

Lecture notes from a series of lectures given by J.P. Murre at the Tata Institute in 1964-65. It is much shorter and accessible than the SGA 1.

J.B. Bost et al
Courbes semi-stables et groupe fondamental en géométrie algébrique (Luminy 1998), J.B. Bost et al. (eds), Progress in Math. **187**, Birkhauser 2000.

Proceedings volume of a series of conferences given at CIRM in 1998. It consists of survey articles introducing both classical and more recent results on the étale fundamental group of curves.

T. Szamuely
Galois Groups and Fundamental Groups, Cambridge Studies in Advanced Mahtematics **117**, Cambridge University Press, 2009.

A pleasant introduction to both Galois and Tannakian categories. It follows a pedestrian path, starting with familiar examples of fundamental groups in number theory and topology before turning progressively - through the enlightening examples of Riemann surfaces and algebraic curves - to the general Grothendieck formalism.

A. Cadoret
Galois categories, in Proceedings of the Summer School "Arithmetic and Geometry around Galois Theory - Istanbul 2009", P. Dèbes et al. (eds), Progress in Math. **304**, Birkhauser, 2012, p. 171–246.

Lecture notes from a Master 2 course given at Bordeaux University during the spring of 2010. It is close in spirit to the notes of J.P. Murre.

Other lecture notes are available online, in particular,
- Tag 0BQ6 of the Stack Project
- H.W Lenstra's lecture notes 'Galois theory for schemes':

http://websites.math.leidenuniv.nl/algebra/GSchemes.pdf

Tannakian categories (F. Ivorra)

N. Saavadra Rivano
Catégories tannakiennes, L.N.M. **265**, Springer, 1972.

The reference book on Tannakian categories but quite monumental too. The following two papers are shorter classical surveys on the subject.

P. Deligne and J.S. Milne
Tannakian categories, in "Hodge cycles, motives and Shimura varieties", L.N.M. **900**, Springer, 1982, pp. 101–228.

P. Deligne
Catégories tannakiennes, in "The Grothendieck Festschrift II", P. Cartier et al. (eds.) , Progress in Math. **87**, Birkhauser, 1990, pp. 111–195.

Y. André
Une introduction aux motifs (motifs purs, motifs mixtes, périodes), Panorama et synthèse **17**, S.M.F. 2004.

An ambitious and bright panoramic survey of the theories of motives.

Mixed motives (J. Ayoub):

C. Mazza, V. Voevodsky and C. Weibel
Lectures on Motivic Cohomology', Clay Mathematics Monographs **2**.

This is a very readable book on mixed motives and motivic cohomology that contains most of the results of the original reference (which is much harder to read):

V. Voevodsky, A. Suslin and E. M. Friedlander
Cycles, Transfers, and Motivic Homology Theories', Annals of Mathematics Studies **143**.

J. Ayoub
L'algbre de Hopf et le groupe de Galois motiviques d'un corps de caractéristique nulle, I & II, Journal fr die reine und angewandte Mathematik (Crelles Journal) **693**, 2014, p. 1–226.

This is the paper where the motivic Hopf algebra and the motivic Galois group are constructed using Voevodsky's motives.

J. Ayoub
Une version relative de la conjecture des périodes de Kontsevich-Zagier" Annals of Mathematics (2) **181**, 2015, p. 905–992.

This paper develops an application of motives and the motivic Galois group to periods in families. An overview of the relation between motives and periods can be found in:

J. Ayoub
Periods and the conjectures of Grothendieck and Kontsevich-Zagier' Newsletter of the E.M.S. **91**, March 2014.

Y. André
Groupes de Galois motiviques et périodes Séminaire Bourbaki, exposé 1104, 2015.

Anabelian geometry (Y. Hoshi)

A.Grothendieck
Esquisse d'un programme (With an English translation on p. 243–283),
In *Geometric Galois actions, 1*, London Math. Soc. L.N.S. **242**, Cambridge Univ. Press, Cambridge, p. 5–48, 1997.

A.Grothendieck
Brief an G. Faltings (With an English translation on pp. 285–293),
In *Geometric Galois actions, 1*, London Math. Soc. L.N.S. **242**, Cambridge Univ. Press, Cambridge, p. 49–58, 1997.

F. Pop
Glimpses of Grothendieck's anabelian geometry,
In *Geometric Galois actions, 1*, London Math. Soc. L.N.S. **242**, Cambridge Univ. Press, Cambridge, p. 113–126, 1997.

H. Nakamura
Galois rigidity of profinite fundamental groups
Sugaku Expositions **10**, no. **2**, p. 195–215, 1997.

G. Faltings
Curves and their fundamental groups (following Grothendieck, Tamagawa and Mochizuki)
Séminaire Bourbaki. Vol. 1997/98. *Astérisque* No. **252**, Exp. No. **840**, **4**, p. 131–150, 1998.

H. Nakamura, A. Tamagawa, and S. Mochizuki
The Grothendieck conjecture on the fundamental groups of algebraic curves
Sugaku Expositions **14**, no. **1**, p. 31–53, 2001.

J. Neukirch, A. Schmidt, and K. Wingberg
Cohomology of number fields. Second edition, G.M.W. **323**. Springer-Verlag, Berlin, 2008.

M. Saïdi and A. Tamagawa
On the anabelian geometry of hyperbolic curves over finite fields
in *Algebraic number theory and related topics 2007, RIMS Kôkyûroku Bessatsu*, **B12**, Res. Inst. Math. Sci. (RIMS), Kyoto, p. 67–89, 2009.

F. Bogomolov and Y. Tschinkel
Introduction to birational anabelian geometry
In *Current developments in algebraic geometry*, Math. Sci. Res. Inst. Publ., **59**, Cambridge Univ. Press, Cambridge, p. 17–63, 2012.

F. Pop
Lectures on anabelian phenomena in geometry and arithmetic
In *Non-abelian fundamental groups and Iwasawa theory*, London Math. Soc. Lecture Note Ser., **393**, Cambridge Univ. Press, Cambridge, p. 1–55, 2012.

J. Stix, *Rational points and arithmetic of fundamental groups. Evidence for the section conjecture*, Lecture Notes in Mathematics, **2054**. Springer, Heidelberg, 2013.

Chtoucas for reductive groups and the geometric Langlands conjecture (S. Morel)

You can find V. Lafforgue's paper on his webpage or on the arXiv.

V. Lafforgue
Chtoucas pour les groupes réductifs et paramétrisation de Langlands globale, preprint.

as well as the introductory versions (both in French and in English):

V. Lafforgue
Introduction aux chtoucas pour les groupes réductifs et la paramétrisation de Langlands globale, preprint.
Introduction to chtoucas for reductive groups and to the global Langlands parameterization, preprint.

There is also also a Bourbaki seminar on the topics:

B. Stroh
La paramétrisation de Langlands globale sur les corps de fonctions, d'après Vincent Lafforgue, Séminaire Bourbaki, exposé 1110, 2016.

The notes of S. Morel's Princeton lectures are temporarily downloadable on the webpage of the conference.

Eventually, for tools involveved in V. Lafforgue's proof, you can also browse:

- (Construction and study of moduli spaces of chtoucas)

  Y. Varshavsky *Moduli spaces of principal F-bundles*, Selecta Math. (N.S.) **10**, p. 131–166, 2004.

- (More elementary stuff about stacks, G-bundles *etc.*) the notes of Gaitsgory's students seminar

  http://www.math.harvard.edu/ ~ gaitsgde/grad_2009/

- (Geometric Satake)

  T. Richarz
  A new approach to the geometric Satake equivalence, Doc. Math. **19**, p. 209–246, 2014.

  D. Gaitsgory
  On de Jong's conjecture, Israel J. Math. **157**, p. 155–191, 2007.

Chabeauty-Kim method (S. Wewers):

M. Kim *The motivic fundamental group of* $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ *and the theorem of Siegel*, Invent. Math. **161**, p. 629–656, 2005.

Kim's seminal paper

M. Hadian-Jazi
Motivic Fundamental Groups and Integral Points, Duke Mathematical Journal **160**, p. 503–565, 2011.

Variations around Kim's method. Probably more accessible than Kim's paper

V. Mc Callum and B. Poonen
The method of Chabauty and Coleman in *Explicit methods in number theory*, Panorama et Synthèses, **36**, S.M.F., p. 99–117, 2012.

An introduction to the p-adic method of Chabauty and Coleman to try and determine explicitly the set of rational points on a genus ≥ 2 projective curve over a number field. Kim's method can be regarded as a wide non-abelian generalization of these ideas.

In case it may help, on

http://www.cmls.polytechnique.fr/perso/cadoret/GTKim.pdf

you can find a synopsis for a working group on Kim's method.